INDIAN STATISTICAL INSTITUTE

CHENNAI CENTRE

M.STAT First Year

2016-17 Semester II

Nonparametric Inference and Sequential Analysis
Final Examination

Total Marks 50. Duration: 3 hours

1. Let X be a non-negative random variable with distribution function $F(.)$. Define the parameter $\theta = \frac{1}{2}\int_0^\infty x(1-F(x))dx$. Find the influence function corresponding to θ. (5)

2. Define runs. Discuss the test for equality of two distributions using run test. Find the null distribution of R, where R is the number of runs. (1+2+3)

3. Discuss the kernel density estimation. Let f be continuous at x and that $h_n \to 0$ and $nh_n \to \infty$ as $n \to \infty$, where h_n is the bin width. Let $\widehat{f}$ be the kernel density estimator of f. Show that $\widehat{f}$ convergence in probability to f. (5)

4. Let $X_1, \ldots, X_n$ be a random sample from a density $f(x-\theta)$, where f is symmetric about zero. We are interested to test the hypothesis $H_0 : \theta = 0$ using sign test and one sample t test. Obtain the asymptotic relative efficiency of the sign test versus t test when the parent population is distributed according to
a) logistic b) normal c) Laplace (8)

5. Explain SPRT. Give an example to show that the sample contains at least one observation less in SPRT than the test based on fixed sample procedure. (2+5)

6. Suppose $X_1, ..., X_n, ...$ are i.i.d. normal random variables with unknown mean μ and variance σ^2. Consider the SPRT for a simple null hypothesis $H_0 : \mu = \mu_0$ versus a simple alternative hypothesis $H_0 : \mu_1 = \mu_1 (> \mu_0)$ with preassigned type I and type II error probabilities α and β, respectively. Write down the expression of Wald's approximation of the ASN and OC function. (7)

7. Suppose that $X_1, ..., X_n, ...$ are i.i.d. random variables having exponential distribution with parameter θ, Find a $100(1-\alpha)\%$ fixed length $(2d)$ confidence interval for θ. (5)

8. Let X_1 and X_2 be independent and identical observation from F and $\mu = E(X)$. Gini index is defined as $\theta = \frac{E|X_1 - X_2|}{2\mu}$. Find the plug in estimator of θ. Construct a $100(1-\alpha)\%$ confidence interval for θ using empirical likelihood method. (2+6)